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POLARIZATION CORRELATIONS IN PAIR PRODUCTION FROM CHARGED AND NEUTRAL STRINGS

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Polarization correlations of e^+e^- pair productions from charged and neutral Nambu strings are investigated, via photon and graviton emissions, respectively and explicit expressions for their corresponding probabilities are derived and found to be *speed* dependent. The strings are taken to be circularly oscillating closed strings, as perhaps the simplest solution of the Nambu action. In the extreme relativistic case, these probabilities coincide, but, in general, are different, and such inquiries, in principle, indicate whether the string is charged or uncharged. It is remarkable that these dynamical relativistic quantum field theory calculations lead to a clear violation of Local Hidden Variables theories.

Keywords: Closed strings; polarization correlations; photons and gravitons exchanges; Bell's inequality; quantum field theory methods.

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We investigate the polarizations correlations of e^+e^- pair productions from charged and neutral Nambu strings via processes of photon and graviton emissions, respectively. We consider circularly oscillating closed strings as a cylindrical symmetric solution[1–8] arising from the Nambu action,[4–12] as perhaps the simplest string structures in field theory studies. Explicit expressions are derived for their corresponding correlations probabilities and are found to be *speed* dependent. In particular, due to the difference of these probabilities, in general, inquiries about such correlations, would indicate whether the string is charged or uncharged. In the extreme relativistic case, however, these probabilities are shown to coincide. The study of such polarization correlations are carried out in the spirit of classic experiments, cf. Refs. 13, 14, to discriminate against Local Hidden Variable (LHV) theories. In this respect alone, it is remarkable that our explicit expressions of polarizations correlations, as obtained from dynamical relativistic quantum field theories, are found to be in clear violation with LHV theories. The speed dependence of polarizations

correlations is a common feature of dynamical computations in quantum field theory[15,16].

The trajectory of the closed string is described by a vector function $\mathbf{R}(\sigma, t)$, where σ parameterizes the string, satisfying[4–12]

$$\ddot{\mathbf{R}} - \mathbf{R}'' = 0, \quad \dot{\mathbf{R}} \cdot \mathbf{R}' = 0, \quad \dot{\mathbf{R}}^2 + \mathbf{R}'^2 = 1, \tag{1}$$

$$\mathbf{R}\left(\sigma + \frac{2\pi}{m}, t\right) = \mathbf{R}(\sigma, t), \tag{2}$$

where the mass scale m is taken to be the mass of the electron, $\dot{\mathbf{R}} = \partial \mathbf{R}/\partial t$, $\mathbf{R}' = \partial \mathbf{R}/\partial \sigma$, with general solution

$$\mathbf{R}(\sigma, t) = \frac{1}{2}\left[\mathbf{A}(\sigma - t) + \mathbf{B}(\sigma + t)\right], \quad \mathbf{A}'^2 = \mathbf{B}'^2 = 1. \tag{3}$$

We consider a solution of the form[2–8]

$$\mathbf{R}(\sigma, t) = \frac{1}{m}\left(\cos m\sigma, \sin m\sigma, 0\right) \sin mt, \tag{4}$$

with the z-axis perpendicular to the plane of oscillations. For a string of total charge Q, this generates a current density[5] $J^\mu(x)$ with structure ($x = (t, \mathbf{r}, z)$)

$$J^\mu(x) = \int \frac{\mathrm{d}^2\mathbf{p}}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{(2\pi)} \int_{-\infty}^{\infty} \frac{\mathrm{d}P^0}{(2\pi)}\, \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{r}} \mathrm{e}^{\mathrm{i}qz} \mathrm{e}^{-\mathrm{i}P^0 t} J^\mu(P^0, \mathbf{p}), \tag{5}$$

$$J^\mu(P^0, \mathbf{p}) = 2\pi \sum_N \delta(P^0 - mN) B^\mu(\mathbf{p}, N), \tag{6}$$

summing over integers,

$$B^0(\mathbf{p}, N) = a_N J^2_{N/2}\left(\frac{|\mathbf{p}|}{2m}\right), \tag{7}$$

$$\mathbf{B}(\mathbf{p}, N) = \frac{mN}{|\mathbf{p}|^2}\mathbf{p} B^0(\mathbf{p}, N), \tag{8}$$

$$a_N = Q(-1)^{N/2} \cos\left(\frac{N\pi}{2}\right), \tag{9}$$

where $J_{N/2}$ are the ordinary Bessel functions of order $N/2$.

We consider the process of e^+e^- pair production via a photon emission, given by the amplitude,[4–8] cf. Refs. 17, 18, up to an overall multiplicative factor irrelevant for the problem at hand.

$$\mathcal{A} \propto J^\mu(2p^0, \mathbf{p}_1 + \mathbf{p}_2) \frac{1}{(p_1 + p_2)^2} \left[\overline{u}(\mathbf{p}_1, \sigma_1)\gamma_\mu v \mathbf{p}_2, \sigma_2)\right] \tag{10}$$

with the four momenta of e^-, e^+, respectively, given by

$$\mathbf{p}_1 = k(0,1,0), \quad \mathbf{p}_2 = k(1,0,0), \quad k = m\gamma\beta, \tag{11}$$

$$p_1^0 = p_2^0 = (\mathbf{k} + m^2)^{1/2} \equiv p^0 = m\gamma, \tag{12}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor. The measurement of the spin projection of the electron is taken along an axis making an angle χ_1 with the z-axis and lying in a plane parallel to the x–z plane,

$$u = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} \xi_1 \\ \\ \dfrac{k\sigma_2}{p^0+m}\xi_1 \end{pmatrix}, \qquad v = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} -\dfrac{k\sigma_1}{p^0+m}\xi_2 \\ \\ \xi_2 \end{pmatrix}, \tag{13}$$

where the direction of the spin of the positron lies in a plane parallel to the y–z plane. For the 2-spinors, we have

$$\xi_1 = \begin{pmatrix} \cos\left(\dfrac{\chi_1}{2}\right) \\ \\ -\sin\left(\dfrac{\chi_1}{2}\right) \end{pmatrix}, \qquad \xi_2 = \begin{pmatrix} \sin\left(\dfrac{\chi_2}{2}\right) \\ \\ \cos\left(\dfrac{\chi_2}{2}\right) \end{pmatrix}. \tag{14}$$

A tedious but straightforward computation gives

$$\mathcal{A} \propto \frac{\left[-\mathrm{i}\left(1-\frac{\gamma\beta^2}{2}\right)\cos\left(\frac{\chi_1-\chi_2}{2}\right) + \left(1+\frac{\gamma\beta^2}{2}\right)\cos\left(\frac{\chi_1+\chi_2}{2}\right)\right]}{(\gamma^2\beta^2-2)} \sum_N \delta(2p^0 - mN)B^0, \tag{15}$$

where we note that $2p^0/m = 2\gamma$ is quantized. *Given* that the above process has occurred, a standard computation, as in Refs. 15, 16, given the following explicit expression for the probability of the simultaneous measurements of the spins of e^-, e^+, with angles χ_1, χ_2, as specified above,

$$P[\chi_1,\chi_2] = \frac{\left(2\sqrt{1-\beta^2}-\beta^2\right)^2\cos^2\left(\frac{\chi_1-\chi_2}{2}\right) + \left(2\sqrt{1-\beta^2}+\beta^2\right)^2\cos^2\left(\frac{\chi_1+\chi_2}{2}\right)}{4\left(2-\beta^2\right)^2} \tag{16}$$

the so-called probability of the polarizations correlations of the emitted pair and is *speed* dependent. If the spin of only one of the particles, say, that of e^-, is measured, then we have to sum (16) over the two possible outcomes for e^+: χ_2, $\chi_2+\pi$, for a given χ_2, i.e., for the probability of measuring the spin of e^- only, we have

$$P[\chi_1,-] = P[\chi_1,\chi_2] + P[\chi_1,\chi_2+\pi] = \frac{1}{2}. \tag{17}$$

Similarly, for the probability $P[-,\chi_2]$, where only a measurement of the spin of e^+ is made, we obtain

$$P[-,\chi_2] = \frac{1}{2}. \tag{18}$$

In the extreme relativistic case $\beta \to 1$, (16) gives

$$P[\chi_1,\chi_2] \longrightarrow \frac{1}{4}\left[\cos^2\left(\frac{\chi_1-\chi_2}{2}\right) + \cos^2\left(\frac{\chi_1+\chi_2}{2}\right)\right]. \tag{19}$$

The neutral string, of a given mass M, generates an energy-momentum tensor density $T^{\mu\nu}(x)$ with structure[6]

$$T^{\mu\nu}(x) = \int \frac{\mathrm{d}^2\mathbf{p}}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{(2\pi)} \int_{-\infty}^{\infty} \frac{\mathrm{d}P^0}{(2\pi)}\, \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{r}} \mathrm{e}^{\mathrm{i}qz} \mathrm{e}^{-\mathrm{i}P^0 t} T^{\mu\nu}(P^0,\mathbf{p}), \tag{20}$$

$$T^{\mu\nu}(P^0,\mathbf{p}) = 2\pi \sum_{N=-\infty}^{\infty} \delta(2p^0 - mN) B^{\mu\nu}(\mathbf{p},N), \tag{21}$$

$$B^{00}(\mathbf{p},N) = \beta_N J^2_{N/2}(z), \quad z = \frac{|\mathbf{p}|}{2m}, \tag{22}$$

$$B^{0a}(\mathbf{p},N) = \beta_N \frac{P^0 p^a}{|\mathbf{p}|^2} J^2_{N/2}(z), \quad a = 1,2 \tag{23}$$

$$B^{ab}(\mathbf{p},N) = \beta_N \left(A_N \delta^{ab} + E_N \frac{p^a p^b}{|\mathbf{p}|^2}\right), \quad a,b = 1,2 \tag{24}$$

$$B^{\mu 3}(\mathbf{p},N) = 0, \quad \mu = 0,1,2,3 \tag{25}$$

$$A_N = \frac{1}{4}\left[J_{\frac{N}{2}+1}(z) - J_{\frac{N}{2}-1}(z)\right]^2, \tag{26}$$

$$E_N = J_{\frac{N}{2}+1}(z) J_{\frac{N}{2}-1}(z), \tag{27}$$

$$\beta_N = M(-1)^{N/2} \cos\left(\frac{N\pi}{2}\right). \tag{28}$$

For e^+e^- pair production via the emission of a graviton, the amplitude of the process is given by

$$\mathcal{A} \propto T^{\sigma\lambda}(2p^0,\mathbf{p}_1+\mathbf{p}_2) \frac{\left[g_{\sigma\mu}g_{\lambda\nu} - \frac{1}{2}g_{\sigma\lambda}g_{\mu\nu}\right]}{(p_1+p_2)^2} T^{\mu\nu}_{e^+e^-}, \tag{29}$$

where $T^{\mu\nu}_{e^+e^-}$ is the energy-momentum tensor density associated with the pair. From (21)–(22), (11), (12), this simplifies to

$$\mathcal{A} \propto \frac{1}{(p_1+p_2)^2}\left\{-2m\overline{u}vT^{00} + 2\left[(\overline{u}\gamma_a v)\left(p^1_b - p^2_b\right) + m\delta_{ab}\overline{u}v\right]T^{ab}\right\}. \tag{30}$$

The recurrence relation

$$J_{\frac{N}{2}-1}(z) = \frac{2\sqrt{2}}{\beta}J_{\frac{N}{2}}(z) - J_{\frac{N}{2}+1}(z), \tag{31}$$

allows one to express A_N, E_N in terms of $J_{N/2}+1$, $J_{N/2}$ which differ by one order only, and for sufficiently high energies, they may be expressed in terms of $J_{N/2}$.

All told, given that the above process has occurred, a direct straightforward simplification of the expression in (30) leads to the following expression for the polarizations correlations probability of the pair

$$P[\chi_1,\chi_2] = \frac{1}{4\left(4-\beta^2-2\sqrt{2}\beta\right)}\left[\left(\sqrt{2(1-\beta^2)}-\sqrt{2}+\beta\right)^2\cos^2\left(\frac{\chi_1-\chi_2}{2}\right) + \left(\sqrt{2(1-\beta^2)}+\sqrt{2}-\beta\right)^2\cos^2\left(\frac{\chi_1+\chi_2}{2}\right)\right] \tag{32}$$

and again is speed dependent, $P[\chi_1,-] = 1/2 = P[-,\chi_2]$ for a measurement of the spin of only one of the particles. The fact that the polarizations correlations probabilities of the e^+e^- pair emitted from the charged and neutral strings are different in general, such inquiries indicate, in principle, whether the string is charged or uncharged. In the extreme relativistic case, the probability in (32) coincides with the expression on the right-hand side of (19) for a charged string.

Finally we note that these dynamical relativistic quantum field theory calculations lead to a violation of LHV theories.[13,14] To this end, define:[13,14]

$$S = P[\chi_1,\chi_2] - P[\chi_1,\chi_2'] + P[\chi_1',\chi_2] + P[\chi_1',\chi_2'] - P[\chi_1',-] - P[-,\chi_2] \tag{33}$$

for two pairs of angles (χ_1,χ_2), (χ_1',χ_2'). To show violation with LHV theories, it is sufficient to consider one experimental situation which gives for S a value outside[13,14] the interval $[-1,0]$. To this end for $\beta = 0.8$, $\chi_1 = 0^\circ$, $\chi_2 = 160^\circ$, $\chi_1' = 100^\circ$, $\chi_2' = 10^\circ$, we obtain $S = -1.088$, $S = -1.103$ for the charged and neutral strings, respectively, leading to a clear violation of LHV theories.

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